Pressure Measurements of a Rotating Liquid for Impulsive Coning Motion

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Nomenclature

a	= radius of the cylinder
A_m	= Fourier coefficient in Eq. (2)
c	= half height of the cylinder
C_p	= pressure coefficient, $(P/\epsilon\rho\dot{\phi}^2 a^2)$
i	= $\sqrt{-1}$
P	= instantaneous pressure
P_{ss}	= steady-state pressure in Eq. (1)
P_{total}	= total pressure in Eq. (1)
$P_{transient}$	= transient pressure in Eq. (1) and defined in Eq. (2)
Re	= Reynolds number, $(a^2\dot{\phi}/\nu)$
T_c	= cone-up time in Eq. (3)
t	= time
T_s	= spin-up time
$\dot{\phi}$	= spin frequency
ϵ	= coning angle
ρ	= liquid density
ν	= liquid kinematic viscosity
τ_m	= eigenfrequency of the transient pressure response
ω	= coning frequency

Abstract

A SPINNING cylinder which contained an incompressible liquid in rigid body rotation was impulsively forced to cone at a small angle away from a vertical orientation. The response of the fluid to this abrupt, circular coning motion was measured by pressure transducers that were located in the end wall of the cylinder. The time for the liquid to achieve a steady-state response to the impulsive coning motion is identified as the cone-up time and was several seconds. Cone-up times were experimentally determined for a range of coning frequencies for a Reynolds number of 5.23×10^5. In general, the cone-up times were comparable to the spin-up time (the time required by a liquid to adjust to an impulsive change in the spin rate of the cylinder when no coning motion is induced). The experimental data indicate that numerical simulations for liquid payloads carried by spin-stabilized, gun launched projectiles must account for both spin up and cone up and that these effects must be treated simultaneously.

Contents

Background

The stability of a spin-stabilized projectile can be reduced markedly by the presence of a liquid payload. This problem has been addressed analytically and experimentally.[1] It can be shown that if certain natural frequencies of the fluid correspond to the fast precessional frequency of the projectile that a periodic liquid moment is applied to the projectile. This moment can be sufficiently large to produce dramatic flight instabilities.

Experiment Description

Pressure measurements for rotating liquids have been made by Aldridge[2] and by Whiting.[3] These investigations considered axisymmetric and nonaxisymmetric disturbances when the liquid was in rigid body rotation or during spin-up. A device used by Whiting was modified to produce impulsive coning motions and is shown in Fig. 1. A rotor is held within a cage support. Below the cage, a small dc motor drives the liquid-filled rotor at constant spin rates (normally 83 Hz). The spin drive motor is surrounded by a smaller cage that is connected to a bushing that is held in a cam. The position of the bushing in the cam can be set and fixed by a pair of adjustable screws, thus producing an inclination of the spin axis of the rotor/cage support assembly to the vertical. The cam is driven from beneath the support stand by a second dc motor via a belt drive, a magnetic clutch, and a pulley. A typical steady coning frequency for this system is 4 Hz. A pair of miniature pressure transducers are mounted on the flat

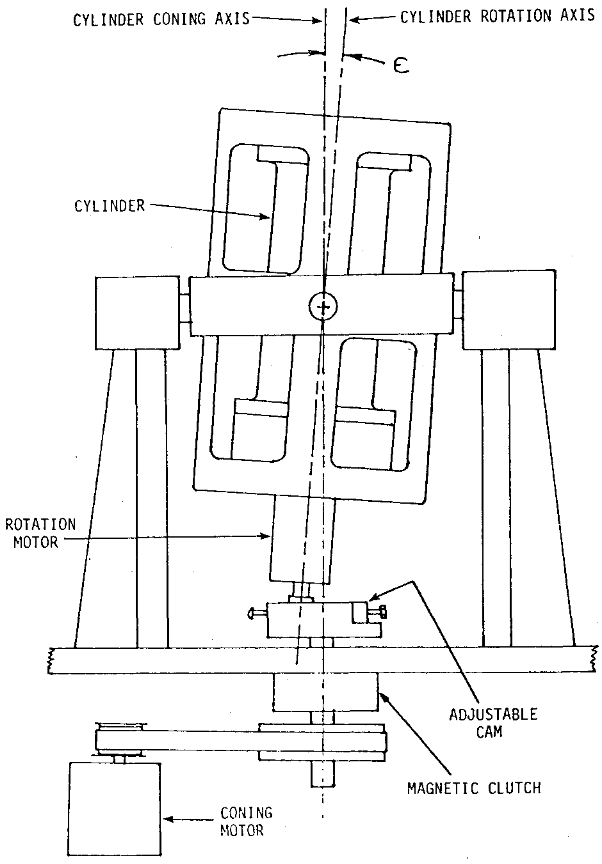

Fig. 1 Experimental device.

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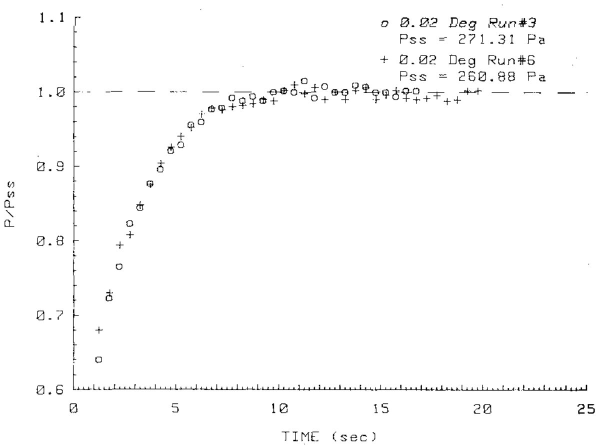

Fig. 2 Pressure vs time for $\omega = 3.918$ Hz and $\epsilon = 0.02$ deg.

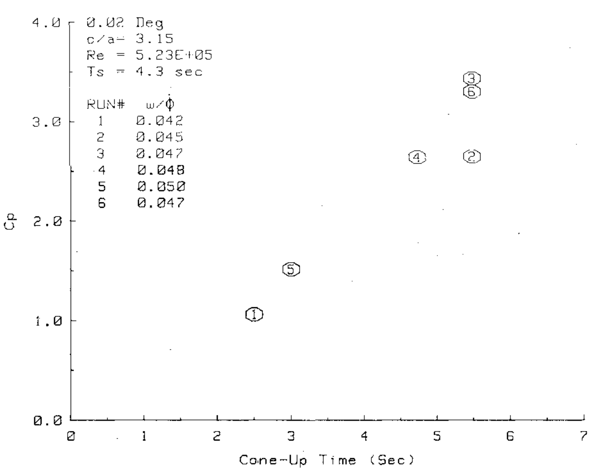

Fig. 3 Cone-up time vs pressure coefficient for $\epsilon = 0.02$ deg.

surface of an insert which forms the end wall of the liquid-filled cylinder. The output from the pressure transducers is amplified, conditioned, and transmitted to the laboratory frame via a telemetry system located below the cylinder and within the rotor.

Assuming that the liquid is in a state of rigid body rotation, any disturbance in the motion of the container will generate a system of inertial waves.[4] If a forced motion of the container is induced and sustained, the wave motion will persist. If the frequency of the forcing motion is programmable, then the pressure of the liquid at the end wall can be measured as a function of the forcing frequency. A pressure coefficient is defined as $C_p = P/\epsilon\rho\dot{\phi}^2 a^2$, where P is the amplitude of the fluctuating part of the pressure.

Pressures were measured during cone-up for ϵ equal to 0.02 and 0.05 deg. A single cylinder geometry [height = 19.99 cm, diameter = 6.35 cm, aspect ratio $(c/a) = 3.15$] was tested at a $Re = 5.23 \times 10^5$ ($\dot{\phi} = 83.3$ Hz, $\nu = 0.01$ cm^2/s). The amplitude of the pressure signal was determined by a spectrum analyzer. A data window of 2.5 s with a flat top passband shape was utilized and resulted in a frequency resolution of 1.45 Hz with an amplitude error of less than 1%. Unsteady pressure data were processed using sliding time samples with overlaps of 0.5 s. Such a technique implicitly assumes that the data are quasisteady during the time interval which is processed, and such an assumption cannot be justified. However, the time to reach a steady-state pressure response can be determined by such a procedure.

Experimental Results

The pressure histories for a transducer located at a radius of 21.2 mm were processed. Figure 2 shows a typical pressure history when the coning frequency equals the fundamental eigenfrequency of the liquid. The pressure histories provide a direct measurement of the time required by the liquid to adjust to the motion of the cylinder. Since P/P_{ss} adjusts to the final steady-state response in an asymptotic form, the time at which $P/P_{ss} = 0.95$ was arbitrarily selected as the cone-up time. The cone-up times were correlated with the steady-state value of C_p (Fig. 3). The cone-up time indicates a linear trend.

A crude estimate of the cone-up time for linear pressure responses can be made. Assume

$$P_{\text{total}} = P_{ss} + P_{\text{transient}} \tag{1}$$

$P_{\text{transient}}$ is the unsteady pressure response during cone-up. Now P_{ss} can be represented as a superposition using the natural modes of oscillation (eigenfrequencies) and a proper set of eigenfunctions. $P_{\text{transient}}$ could be expressed in the same fashion. When the coning frequency is equal to τ_m, the expansion for $P_{\text{transient}}$ will be dominated by the single term involving τ_m or

$$P_{\text{transient}} = \Sigma A_m e^{i\dot{\phi}\tau_m t} \tag{2}$$

The eigenfrequency τ_m is complex and the imaginary part is $0\ (Re^{1/2})$ and controls the decay time for $P_{\text{transient}}$. The viscous corrected eigenfrequency can be computed by methods established by Wedemeyer.[5] The cone-up time T_c, where linear pressure responses occur, is approximately

$$T_c = Re^{-1/2}/\dot{\phi} \tag{3}$$

For $\epsilon = 0.02$ deg and $\tau_m = 0.467 + 1.35 \times 10^{-3}$, then $T_c =$ 1.41 s. This estimate of cone-up time should correspond to the e-folding time or the time for P/P_{ss} to reach 0.623. $P/P_{ss} = 0.623$ occurs at approximately 1 s, which roughly corresponds to the estimated value for T_c. Further experiments to determine the dependence of T_c on ϵ and Re should be made.

There is, however, an overlap of the spin-up and cone-up regimes. An estimate for the spin-up time when $Re \gg 1$ is $T_s = (c/a)Re^{1/2}/\dot{\phi}$. Since spin-up is an exponential process, T_s represents the characteristic or e-folding time for spin-up.[4] Since both T_s and T_c are $0(Re^{1/2})$, the unsteady processes for cone-up and spin-up must be considered simultaneously for a liquid-filled projectile. For the present experiments, $T_s = 4.3$ s (as labeled on Fig. 3).

Conclusions

A series of experiments were performed to determine the response time of a spinning liquid to impulsive coning motion. For the coning angle and Reynolds number tested, the cone-up times were several seconds in duration. Linear correlations exist between the cone-up times and steady-state pressure coefficients. Coning angles were restricted to very small angles (0.02 and 0.05 deg) so that the pressure data could be interpreted easily without the presence of gross nonlinear effects. It is probable that yaw levels typical to real projectiles will modify the conclusions that have been reached by this initial investigation, but an engineering estimate for the cone-up time is $Re^{1/2}/\dot{\phi}$.

References

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